F-BF Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. ★

F-LE Linear, Quadratic, and Exponential Models ★

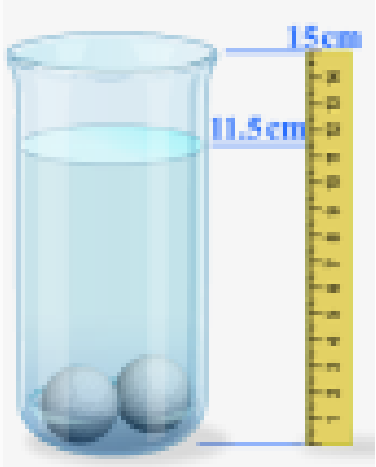
Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Tom is doing an experiment adding golf balls to a glass jar containing water. The picture and the table show what happens to the height of the water as Tom adds golf balls.



Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8

Part a:

Use the following options to complete the sentences and the equation below based on the results of Tom's experiment.

golf balls	change	glass jars	water height	1.16
1.2	1.3	9.0	12.0	13.8

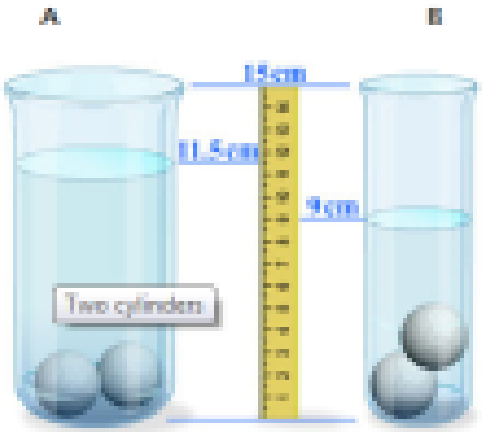
The height of the water changes at an average rate of about _____centimeters per golf ball. If these data were graphed with the number of golf balls as their independent variable, the y-intercept for the graph would be about _____centimeters. This means that for zero _____, the _____is 9 centimeters. Tom's table and graph can be represented by the trend line with the equation y = _____x+_____.

Part b:
There are several ways that Tom could modify the conditions of his experiment. What modifications would increase the rate of change in the height of the water level with respect to the number of golf balls? Select all that apply.

- ☐ Use larger golf balls
- ☐ Decrease the diameter of the glass jar
- ☐ Drop the golf balls into the glass jar at a faster rate
- ☐ Add 5 cm of water to the glass jar
- ☐ Drop the golf balls into the glass jar two at a time

Tom repeats his experiment with a different glass jar. The new glass jar, B, has a smaller radius than the original glass jar, A.

Data from Experiment with Glass Jar A	
Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8



Tom forgot to write down the initial height of the water in glass jar B, but he measured the water height at 9 centimeters after adding two golf balls.

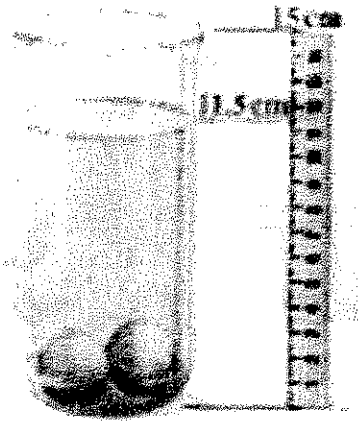
Question a: When Tom creates graphs of the data from both experiments, how will the y-intercepts of the graphs be different for glass jar A versus glass jar B? Explain how you know.

Question b: How will the rate of change in the experiment using glass jar B be different than the rate of change in the experiment using glass jar A? Explain how you know.

Question c: Suppose glass jar B has a water height of 5 centimeters with no golf balls, and the water height increases at a rate of 2 centimeters per golf ball added. Tom continues to add golf balls to each glass jar. He discovers that there is a number of golf balls at which the height of the water in each glass jar is the same. How many golf balls will be in each jar when the water in each reaches the same height?

Sample A

Tom is doing an experiment adding golf balls to a glass jar containing water. The picture and the table show what happens to the height of the water as Tom adds golf balls.



Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8

+1.2
+1.3
+1.2
+1.1
1.2

Part a:

Use the following options to complete the sentences and the equation below based on the results of Tom's experiment.

golf balls	change	glass jars	water height	1.16
1.2	1.3	9.0	12.0	13.8

The height of the water changes at an average rate of about 1.2 centimeters per golf ball. If these data were graphed with the number of golf balls as their independent variable, the y-intercept for the graph would be about 9.0 centimeters. This means that for zero golf balls, the water height is 9 centimeters. Tom's table and graph can be represented by the trend line with the equation $y =$ 1.2 $x +$ 9.0.

Part b:

There are several ways that Tom could modify the conditions of his experiment. What modifications would increase the rate of change in the height of the water level with respect to the number of golf balls? Select all that apply.

- [x] Use larger golf balls
- [x] Decrease the diameter of the glass jar
- [ ] Drop the golf balls into the glass jar at a faster rate
- [ ] Add 5 cm of water to the glass jar
- [ ] Drop the golf balls into the glass jar two at a time

Sample A

Tom repeats his experiment with a different glass jar. The new glass jar, B, has a smaller radius than the original glass jar, A.

Data from Experiment with Glass Jar A	
Number of golf balls, x	Height of water in centimeters, y
0	9.0
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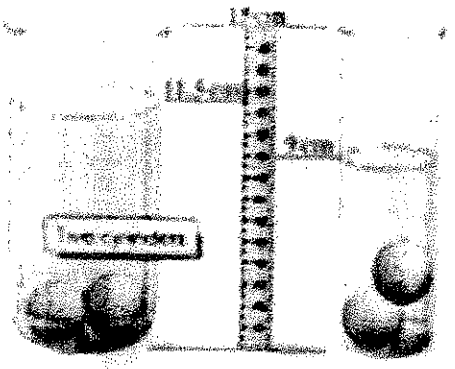


Tom forgot to write down the initial height of the water in glass jar B, but he measured the water height at 9 centimeters after adding two golf balls.

Question a: When Tom creates graphs of the data from both experiments, how will the y-intercepts of the graphs be different for glass jar A versus glass jar B? Explain how you know.

The y-intercept of jar B will be less than jar A since jar A start at 9cm with 0 golf balls while jar B was at 9cm with 2 golf balls.

Question b: How will the rate of change in the experiment using glass jar B be different than the rate of change in the experiment using glass jar A? Explain how you know.

The rate of change will be higher since the radius of jar B is small than jar A.

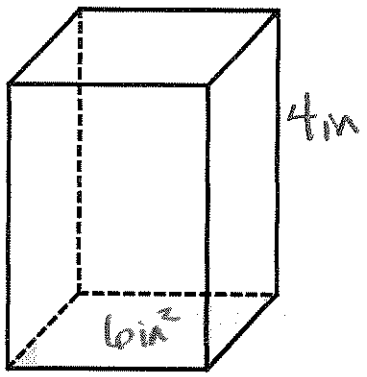
Question c: Suppose glass jar B has a water height of 5 centimeters with no golf balls, and the water height increases at a rate of 2 centimeters per golf ball added. Tom continues to add golf balls to each glass jar. He discovers that there is a number of golf balls at which the height of the water in each glass jar is the same. How many golf balls will be in each jar when the water in each reaches the same height?

$2x+5 = 1.2x+9$

5 golf balls

Fill It Up, Please – Part I (Option B)

1. Box A has a height of 4 inches and a base area of 6 square inches. Sandy is filling the box with sand using a scoop that holds 3 cubic inches. To be precise in her measurements, she only adds whole scoops of sand. After each scoop is added, she levels the sand in the box and measures the height of the sand.



Box A

a. What is the equation for the height of a rectangular prism, h, in terms of the volume, V, and the base area, B?

$\frac{V}{B} = \frac{Bh}{B}$

$\frac{V}{B} = h$

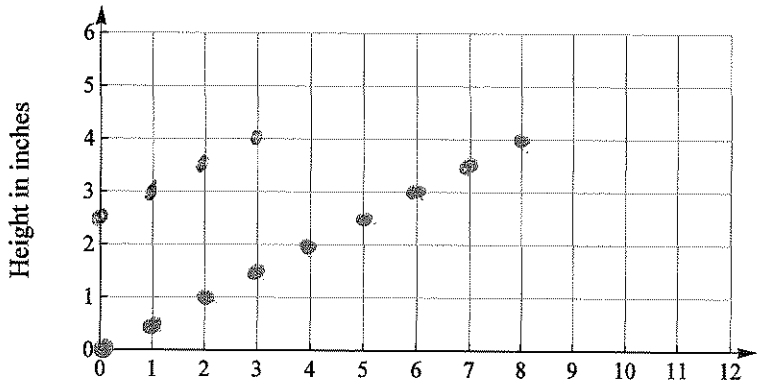
$h = \frac{V}{B}$

b. Complete the table that indicates the height of the sand, h, in Box A for each scoop of sand, s.

Total number of scoops, s	Process column for the height of the sand	Current height of the sand, h
1 scoop	$h = \frac{1 \text{ scoop}\left(\frac{3 \text{ in.}^3}{\text{scoop}}\right)}{6 \text{ in.}^2}$	$\frac{1}{2}$ in.
3 scoops	$\frac{(3 \text{ scoops})\left(\frac{3 \text{in}^3}{\text{scoop}}\right)}{6 \text{in}^2}$	$\frac{9}{6} = \frac{3}{2}$ in.
4 scoops	$\frac{(4)(3)}{6}$	$\frac{12}{6} = 2$ in
8 scoops	$\frac{(8)(3)}{6}$	4 in

c. Plot the ordered pairs, (s, h), from the table and fill in the additional points that were not determined in the table that indicate that Sandy is filling the box from empty to full.

Box A



d. How many scoops are required to fill Box A?

8

e. What is the domain of the graph of height of the sand, h, versus the number of scoops, s? Is the domain discrete or continuous? Explain the answers in terms of the situation.

$0 \le s \le 8$ where s is whole numbers

Discrete domain b/c only full scoops are used.

f. What is the range of the graph of height of the sand, h, versus the number of scoops, s? Is the range discrete or continuous? Explain the answers in terms of the situation.

~~$0 \le h \le 4$~~ $0, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4$

Discrete range height jumps $\frac{1}{2}$ inch with each scoop.

g. What is the y-intercept of the graph of height versus the number of scoops? Explain the answer in terms of the situation.

y-int = 0

When you start there is no sand in the box.

h. What is the rate of change in the height of the sand with respect to the number of scoops? Show how to calculate the answer using two points. Include units of measure in your work and in your answers.

(0,0)
(2,1)

$\frac{1-0 \text{ inch}}{2-0 \text{ scoops}} = \frac{1}{2}\frac{\text{inch}}{\text{scoop}}$

i. What is the equation for the height of the sand in the box, h, in terms of the number of scoops of sand, s? Include the domain in the answer.

$y = mx + b$ $y = \frac{1}{2}x + 0$ $h = \frac{1}{2}s$ Domain $0 \le s \le 8$

j. Suppose Box A initially contained 15 cubic inches of sand.

- What is the initial height of the sand? Indicate units of measure in your work and answers.

$V=Bh$ $V=15in^3$ $15=6h$

$B=6in^2$ $h=\frac{15in^3}{6in^2}$ $\frac{5}{2}$ in

- What features of the graph will be affected by this new initial value?

Start at $y=\frac{5}{2}$ and fewer points on the graph

- Using a different color pencil, graph the data on the grid provided in part (c).

- Use the options in the cells of the table to complete the sentences provided. Some may be used more than once while others may not be used at all.

0	height	$0 \le s \le 3$	$\left\{\frac{5}{2}, 3, \frac{7}{2}, 4\right\}$
$\frac{1}{2}$	volume	$0 \le s \le 4$	$\left\{15, 15\frac{1}{2}, 16, 16\frac{1}{2}\right\}$
$\frac{5}{2}$	real	$0 \le s \le 15$	$\left\{15, 15\frac{1}{2} \ldots 21\frac{1}{2}, 22\right\}$
3	whole	$0 \le h \le 3$	$h=\frac{1}{2}s$
4	discrete	$0 \le h \le 4$	$h=\frac{1}{2}s+\frac{5}{2}$
15	continuous	$0 \le h \le 15$	$h=\frac{1}{2}s+15$

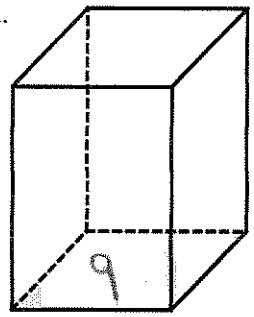
The y-intercept is $\frac{5}{2}$ which means that for 0 scoops of sand, the height of the sand is $\frac{5}{2}$ inches. The height of the sand in the box changes at a rate of $\frac{1}{2}$ inch per scoop which means that for every scoop of sand that is added to the box, the height increases by $\frac{1}{2}$ inch.

The data are discrete because Sandy adds whole scoops. The height of the sand can be modeled by the equation $h=\frac{1}{2}s+\frac{5}{2}$ where the restrictions on the domain require that $0 \le s \le 3$ and $s \in$ whole numbers since 3 scoops are required to completely fill the box. Since the box initially contains sand to a height of $\frac{5}{2}$ inches and will hold sand to a height of 4 inches, and adding one whole scoop increases the height by $\frac{1}{2}$ inch, the range is $\left\{\frac{5}{2}, 3, \frac{7}{2}, 4\right\}$

k. Which of the following remain constant as Sandy fills Box A with sand as described? Mark all correct choices.

- ☑ The base area of the box
- ☑ The height of the box
- ☑ The volume of the scoop
- ☐ The height of the sand
- ☑ The base area of the prism formed by the sand
- ☑ The cross-sectional area of the sand parallel to the base after the sand is leveled
- ☑ The rate of change of the height of the sand with respect to the number of scoops

2. Box B has a height of 4 inches and a base area of 9 square inches. Sandy is filling the box with sand using a scoop that holds 3 cubic inches. To be precise in her measurements, she only adds whole scoops of sand. After each scoop is added, she levels the sand in the box and measures the height of the sand.



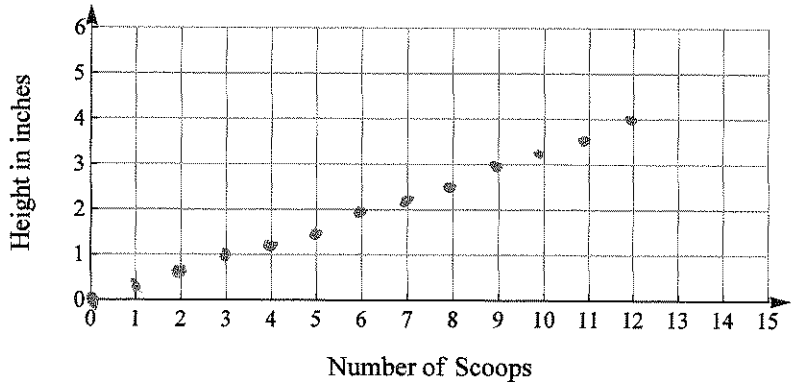
Box B

a. Complete the table that indicates the height of the sand in the box after each scoop is added.

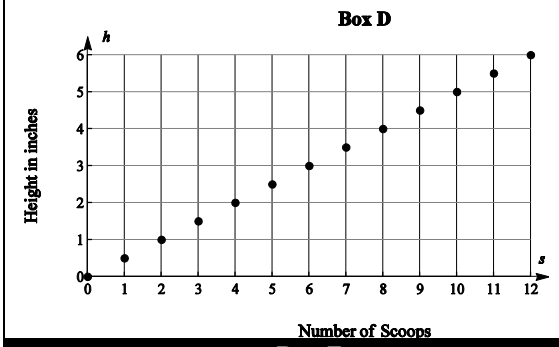
Total number of scoops, s	Current height of the sand, h
3 scoops	1 in.
5 scoops	$\frac{5}{3}$ in
8 scoops	$\frac{8}{3}$ in
10 scoops	$\frac{10}{3}$ in

b. Plot the ordered pairs, (s, h), from the table and fill in the additional points that were not determined in the table that indicate that Sandy is filling the box from empty to full.

Box B



7. Sandy has four empty boxes whose base areas are unknown. The volume of the scoops she is using to fill the boxes is also unknown. To be precise in her measurements, she adds whole scoops of sand, levels the sand in the box, and measures the height of the sand. Information about Box D, Box E, Box F, and Box G is provided in the following table

Box D	Box E:
	<table><tr><th>Total number of scoops, s</th><th>Current height of the sand, h</th></tr><tr><td>6 scoops</td><td>2 in.</td></tr><tr><td>8 scoops</td><td>$2\frac{2}{3}$ in.</td></tr></table>
Box F	**Box G**
$h = \frac{2}{3}s,\ 0 \le s \le 4$ and $s \in$ whole numbers, where h is the height of the sand in inches and s is the number of scoops of sand	Box G has a height of 6 inches and is full of sand when 8 scoops of sand are added.

a. Based on the information given in the previous table, determine the rates of change of the height of the sand in Box D, Box E, Box F, and Box G with respect to the number of scoops of sand. In the following table, record your work and explain how to determine each answer in words or show your numerical work. Include units in your work and answers.

Box D	Box E:
Box F	**Box G**

b. Write an inequality statement comparing the rates of change in the height of the sand in the box with respect to the number of scoops of sand for Box D, Box E, Box F, and Box G.

Note: This prototype task is from http://www.parcconline.org.

Part a:

A car magazine is writing a story about four cars. For each car, they will report the number of miles driven for different amounts of gas.

This table shows the number of miles driven by Car A for different amounts of gas. If Car A uses gas at a constant rate, fill in the blanks to complete the table.

Miles driven		360	480	
Gallons of gas	10	15		24

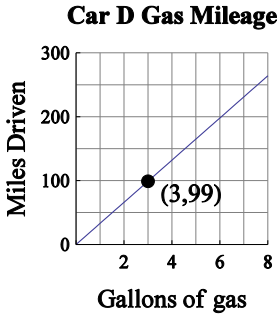
Part b:

The magazine received gas mileage information for cars from several companies.

Miles driven		360	480	
Gallons of gas	10	15		24

$D = 18g$ where D represents the distance traveled in miles, and g represents gallons of gas consumed.

Car C can travel 324 miles on a 12-gallon tank.



The magazine will list the cars in order. Drag the tiles to show the order of the four cars from **greatest** number of miles per gallon to **least** number of miles per gallon.

1st Place	2nd Place	3rd Place	4th Place

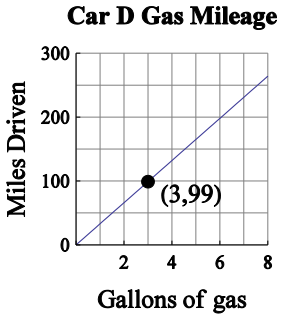
Part c:

The magazine received gas mileage information for cars from several companies.

Miles driven		360	480	
Gallons of gas	10	15		24

$D = 18g$ where D represents the distance traveled in miles, and g represents gallons of gas consumed.

Car C can travel 324 miles on a 12-gallon tank.



Krystal bought one of these cars. She drove 924 miles and used 28 gallons of gas. Based on her gas consumption, which car did she most likely buy?

☐ Car A ☐ Car B

☐ Car C ☐ Car D

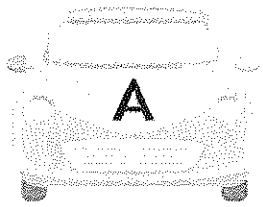
Part a:

A car magazine is writing a story about four cars. For each car, they will report the number of miles driven for different amounts of gas.

This table shows the number of miles driven by Car A for different amounts of gas. If Car A uses gas at a constant rate, fill in the blanks to complete the table.

24 m per gallon

Miles driven	240	360	480	576
Gallons of gas	10	15	20	24

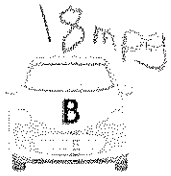


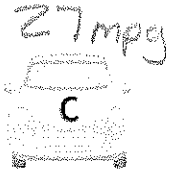
24 x 10 = 240

24 + 24 = 576

Part b:

The magazine received gas mileage information for cars from several companies.



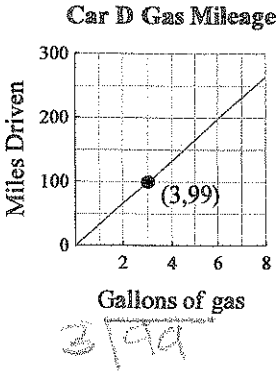


Miles driven	240	360	480	576
Gallons of gas	10	15	20	24

$D = 18g$ where D represents the distance traveled in miles, and g represents gallons of gas consumed.

Car C can travel 324 miles on a 12-gallon tank.

27
12 | 324



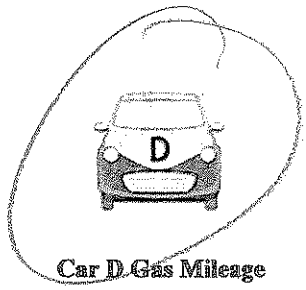
3 | 99

The magazine will list the cars in order. Drag the tiles to show the order of the four cars from **greatest** number of miles per gallon to **least** number of miles per gallon.

1st Place	2nd Place	3rd Place	4th Place
D	C	A	B

Part c:

The magazine received gas mileage information for cars from several companies.



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Gallons of gas	10	15		24

$D = 18g$ where D represents the distance traveled in miles, and g represents gallons of gas consumed.

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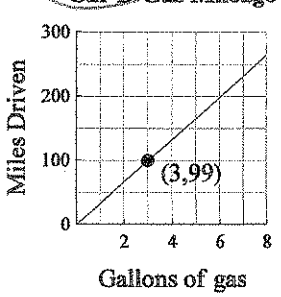

Krystal bought one of these cars. She drove 924 miles and used 28 gallons of gas. Based on her gas consumption, which car did she most likely buy?

☐ Car A ☐ Car B

☐ Car C ☑ Car D

33
28 | 924